

Rational Numbers

MODULE



3



ESSENTIAL QUESTION

How can you use rational numbers to solve real-world problems?



Real-World Video

In many competitive sports, scores are given as decimals. For some events, the judges' scores are averaged to give the athlete's final score.

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LESSON 3.1

Rational Numbers and Decimals



FL 7.NS.1.2b, 7.NS.1.2d

LESSON 3.2

Adding Rational Numbers



FL 7.NS.1.1a, 7.NS.1.1b, 7.NS.1.1d, 7.NS.1.3

LESSON 3.3

Subtracting Rational Numbers



FL 7.NS.1.1, 7.NS.1.1c

LESSON 3.4

Multiplying Rational Numbers



FL 7.NS.1.2, 7.NS.1.2a, 7.NS.1.2c

LESSON 3.5

Dividing Rational Numbers



FL 7.NS.1.2, 7.NS.1.2b, 7.NS.1.2c

LESSON 3.6

Applying Rational Number Operations



FL 7.NS.1.3, 7.EE.2.3

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Animated Math

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Are YOU Ready?

Complete these exercises to review skills you will need for this module.



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Multiply Fractions

EXAMPLE $\frac{3}{8} \times \frac{4}{9}$ $\frac{3}{8} \times \frac{4}{9} = \frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{8}}} \times \frac{\overset{1}{\cancel{4}}}{\underset{3}{\cancel{9}}}$ *Divide by the common factors.*
 $= \frac{1}{6}$ *Simplify.*

Multiply. Write the product in simplest form.

1. $\frac{9}{14} \times \frac{7}{6}$ _____ 2. $\frac{3}{5} \times \frac{4}{7}$ _____ 3. $\frac{11}{8} \times \frac{10}{33}$ _____ 4. $\frac{4}{9} \times 3$ _____

Operations with Fractions

EXAMPLE $\frac{2}{5} \div \frac{7}{10} = \frac{2}{5} \times \frac{10}{7}$ *Multiply by the reciprocal of the divisor.*
 $= \frac{\overset{2}{\cancel{5}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{2}{\cancel{10}}}{\underset{7}{\cancel{10}}}$ *Divide by the common factors.*
 $= \frac{4}{7}$ *Simplify.*

Divide.

5. $\frac{1}{2} \div \frac{1}{4}$ _____ 6. $\frac{3}{8} \div \frac{13}{16}$ _____ 7. $\frac{2}{5} \div \frac{14}{15}$ _____ 8. $\frac{4}{9} \div \frac{16}{27}$ _____
9. $\frac{3}{5} \div \frac{5}{6}$ _____ 10. $\frac{1}{4} \div \frac{23}{24}$ _____ 11. $6 \div \frac{3}{5}$ _____ 12. $\frac{4}{5} \div 10$ _____

Order of Operations

EXAMPLE $50 - 3(3 + 1)^2$ *To evaluate, first operate within parentheses.*
 $50 - 3(4)^2$ *Next simplify exponents.*
 $50 - 3(16)$ *Then multiply and divide from left to right.*
 $50 - 48$ *Finally add and subtract from left to right.*
2

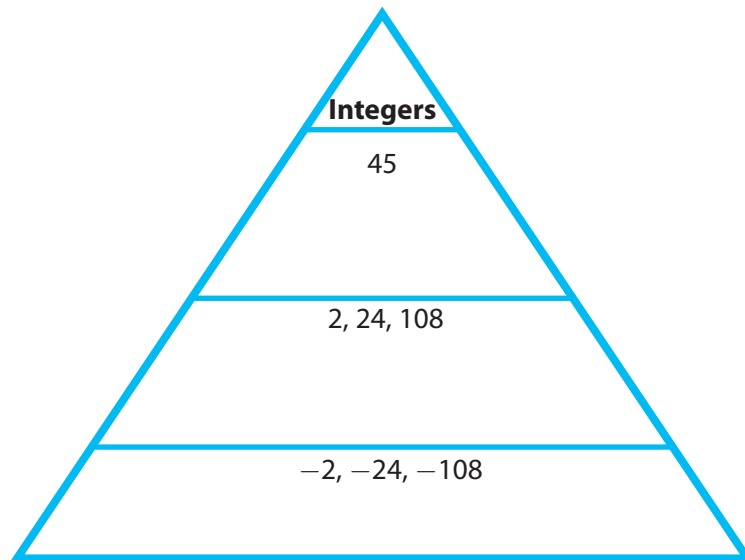
Evaluate each expression.

13. $21 - 6 \div 3$ _____ 14. $18 + (7 - 4) \times 3$ _____ 15. $5 + (8 - 3)^2$ _____
16. $9 + 18 \div 3 + 10$ _____ 17. $60 - (3 - 1)^4 \times 3$ _____ 18. $10 - 16 \div 4 \times 2 + 6$ _____

Reading Start-Up

Visualize Vocabulary

Use the ✓ words to complete the graphic. You can put more than one word in each section of the triangle.



Vocabulary

Review Words

- integers (*enteros*)
- ✓ negative numbers (*números negativos*)
- pattern (*patrón*)
- ✓ positive numbers (*números positivos*)
- ✓ whole numbers (*números enteros*)

Preview Words

- additive inverse (*inverso aditivo*)
- opposite (*opuesto*)
- rational number (*número racional*)
- repeating decimal (*decimal periódico*)
- terminating decimal (*decimal finito*)

Understand Vocabulary

Complete the sentences using the preview words.

1. A decimal number for which the decimals come to an end is a _____ decimal.
2. The _____, or _____, of a number is the same distance from 0 on a number line as the original number, but on the other side of 0.

Active Reading

Layered Book Before beginning the module, create a layered book to help you learn the concepts in this module. At the top of the first flap, write the title of the module, "Rational Numbers." Label the other flaps "Adding," "Subtracting," "Multiplying," and "Dividing." As you study each lesson, write important ideas, such as vocabulary and processes, on the appropriate flap.





MODULE 3

Unpacking the Standards

Understanding the Standards and the vocabulary terms in the Standards will help you know exactly what you are expected to learn in this module.

FL 7.NS.1.3

Solve real-world and mathematical problems involving the four operations with rational numbers.

Key Vocabulary

rational number (*número racional*)

Any number that can be expressed as a ratio of two integers.

What It Means to You

You will add, subtract, multiply, and divide rational numbers.

UNPACKING EXAMPLE 7.NS.1.3

$$-15 \cdot \frac{2}{3} - 12 \div 1\frac{1}{3}$$

$$-\frac{15}{1} \cdot \frac{2}{3} - \frac{12}{1} \div \frac{4}{3}$$

$$-\frac{15}{1} \cdot \frac{2}{3} - \frac{12}{1} \cdot \frac{3}{4}$$

$$-\frac{\overset{5}{\cancel{15}} \cdot 2}{\underset{1}{\cancel{3}}} - \frac{\overset{3}{\cancel{12}} \cdot 3}{\underset{1}{\cancel{4}}}$$

$$-\frac{10}{1} - \frac{9}{1} = -10 - 9 = -19$$

Write as fractions.

To divide, multiply by the reciprocal.

Simplify.

Multiply.

FL 7.NS.1.3

Solve real-world and mathematical problems involving the four operations with rational numbers.

What It Means to You

You will solve real-world and mathematical problems involving the four operations with rational numbers.

UNPACKING EXAMPLE 7.NS.1.3

In 1954, the Sunshine Skyway Bridge toll for a car was \$1.75. In 2012, the toll was $\frac{5}{7}$ of the toll in 1954. What was the toll in 2012?

$$1.75 \cdot \frac{5}{7} = 1\frac{3}{4} \cdot \frac{5}{7}$$

$$= \frac{7}{4} \cdot \frac{5}{7}$$

$$= \frac{\overset{1}{\cancel{7}} \cdot 5}{4 \cdot \underset{1}{\cancel{7}}}$$

$$= \frac{5}{4} = 1.25$$

Write the decimal as a fraction.

Write the mixed number as an improper fraction.

Simplify.

Multiply, then write as a decimal.

The Sunshine Skyway Bridge toll for a car was \$1.25 in 2012.



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LESSON 3.1 Rational Numbers and Decimals

 **FL** 7.NS.1.2d

Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. Also 7.NS.1.2b



ESSENTIAL QUESTION

How can you convert a rational number to a decimal?

EXPLORE ACTIVITY

 **FL** 7.NS.1.2b, 7.NS.1.2d

Describing Decimal Forms of Rational Numbers

A **rational number** is a number that can be written as a ratio of two integers a and b , where b is not zero. For example, $\frac{4}{7}$ is a rational number, as is 0.37 because it can be written as the fraction $\frac{37}{100}$.

- A** Use a calculator to find the equivalent decimal form of each fraction. Remember that numbers that repeat can be written as 0.333... or $0.\overline{3}$.

Fraction	$\frac{1}{4}$	$\frac{5}{8}$	$\frac{2}{3}$	$\frac{2}{9}$	$\frac{12}{5}$		
Decimal Equivalent						0.2	0.875

- B** Now find the corresponding fraction of the decimal equivalents given in the last two columns in the table. Write the fractions in simplest form.
- C Conjecture** What do you notice about the digits after the decimal point in the decimal forms of the fractions? Compare notes with your neighbor and refine your conjecture if necessary.

Reflect

1. Consider the decimal 0.101001000100001000001.... Do you think this decimal represents a rational number? Why or why not?

2. Do you think a negative sign affects whether or not a number is a rational number? Use $-\frac{8}{5}$ as an example.

EXPLORE ACTIVITY (cont'd)

3. Do you think a mixed number is a rational number? Explain.



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Writing Rational Numbers as Decimals

You can convert a rational number to a decimal using long division. Some decimals are **terminating decimals** because the decimals come to an end. Other decimals are **repeating decimals** because one or more digits repeat infinitely.

EXAMPLE 1



FL 7.NS.1.2d

Write each rational number as a decimal.

A $\frac{5}{16}$

Divide 5 by 16.
Add a zero after the decimal point.
Subtract 48 from 50.
Use the grid to help you complete the long division.

Add zeros in the dividend and continue dividing until the remainder is 0.

The decimal equivalent of $\frac{5}{16}$ is 0.3125.

		0.	3	1	2	5	
1	6)	5.	0	0	0	0
			-4	8			
				2	0		
				-1	6		
					4	0	
					-3	2	
						8	0
						-8	0
							0

B $\frac{13}{33}$

Divide 13 by 33.
Add a zero after the decimal point.
Subtract 99 from 130.
Use the grid to help you complete the long division.

You can stop dividing once you discover a repeating pattern in the quotient.

Write the quotient with its repeating pattern and indicate that the repeating numbers continue.

The decimal equivalent of $\frac{13}{33}$ is 0.3939..., or $0.\overline{39}$.

		0.	3	9	3	9	
3	3)	1	3.	0	0	0
			-9	9			
				3	1	0	
				-2	9	7	
					1	3	0
					-9	9	
						3	1
						-2	9
							7
							1
							3

Math Talk

Mathematical Practices

Do you think that decimals that have repeating patterns always have the same number of digits in their pattern? Explain.

YOUR TURN

Write each rational number as a decimal.

4. $\frac{4}{7}$ _____

5. $\frac{1}{3}$ _____

6. $\frac{9}{20}$ _____

Writing Mixed Numbers as Decimals

You can convert a mixed number to a decimal by rewriting the fractional part of the number as a decimal.

EXAMPLE 2



FL 7.NS.1.2d

Shawn rode his bike $6\frac{3}{4}$ miles to the science museum. Write $6\frac{3}{4}$ as a decimal.

STEP 1

Rewrite the fractional part of the number as a decimal.

$$\begin{array}{r} 0.75 \\ 4 \overline{)3.00} \\ -28 \\ \hline 20 \\ -20 \\ \hline 0 \end{array}$$

Divide the numerator by the denominator.



$6\frac{3}{4}$ mi



STEP 2

Rewrite the mixed number as the sum of the whole part and the decimal part.

$$\begin{aligned} 6\frac{3}{4} &= 6 + \frac{3}{4} \\ &= 6 + 0.75 \\ &= 6.75 \end{aligned}$$

YOUR TURN

7. Yvonne made $2\frac{3}{4}$ quarts of punch. Write $2\frac{3}{4}$ as a decimal. $2\frac{3}{4} =$ _____

Is the decimal equivalent a terminating or repeating decimal

8. Yvonne bought a watermelon that weighed $7\frac{1}{3}$ pounds. Write $7\frac{1}{3}$ as a decimal. $7\frac{1}{3} =$ _____

Is the decimal equivalent a terminating or repeating decimal?



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Guided Practice

Write each rational number as a decimal. Then tell whether each decimal is a terminating or a repeating decimal. (Explore Activity and Example 1)

1. $\frac{3}{5} =$ _____

2. $\frac{89}{100} =$ _____

3. $\frac{4}{12} =$ _____

4. $\frac{25}{99} =$ _____

5. $\frac{7}{9} =$ _____

6. $\frac{9}{25} =$ _____

7. $\frac{1}{25} =$ _____

8. $\frac{25}{176} =$ _____

9. $\frac{12}{1,000} =$ _____

Write each mixed number as a decimal. (Example 2)

10. $11\frac{1}{6} =$ _____

11. $2\frac{9}{10} =$ _____

12. $8\frac{23}{100} =$ _____

13. $7\frac{3}{15} =$ _____

14. $54\frac{3}{11} =$ _____

15. $3\frac{1}{18} =$ _____

16. Maggie bought $3\frac{2}{3}$ lb of apples to make some apple pies. What is the weight of the apples written as a decimal? (Example 2)

$3\frac{2}{3} =$ _____

17. Harry's dog weighs $12\frac{7}{8}$ pounds. What is the weight of Harry's dog written as a decimal? (Example 2)

$12\frac{7}{8} =$ _____



ESSENTIAL QUESTION CHECK-IN

18. Tom is trying to write $\frac{3}{47}$ as a decimal. He used long division and divided until he got the quotient 0.0638297872, at which point he stopped. Since the decimal doesn't seem to terminate or repeat, he concluded that $\frac{3}{47}$ is not rational. Do you agree or disagree? Why?

3.1 Independent Practice



FL 7.NS.1.2b, 7.NS.1.2d



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Team Sports

Sport	Number of Players
Baseball	9
Basketball	5
Football	11
Hockey	6
Lacrosse	10
Polo	4
Rugby	15
Soccer	11



Use the table for 19–23. Write each ratio in the form $\frac{a}{b}$ and then as a decimal. Tell whether each decimal is a terminating or a repeating decimal.

19. basketball players to football players

20. hockey players to lacrosse players

21. polo players to football players

22. lacrosse players to rugby players

23. football players to soccer players

24. **Look for a Pattern** Beth said that the ratio of the number of players in any sport to the number of players on a lacrosse team must always be a terminating decimal. Do you agree or disagree? Why?

25. Yvonne bought $4\frac{7}{8}$ yards of material to make a dress.

a. What is $4\frac{7}{8}$ written as an improper fraction? _____

b. What is $4\frac{7}{8}$ written as a decimal? _____

c. **Communicate Mathematical Ideas** If Yvonne wanted to make 3 dresses that use $4\frac{7}{8}$ yd of fabric each, explain how she could use estimation to make sure she has enough fabric for all of them.

- 26. Vocabulary** A rational number can be written as the ratio of one _____ to another and can be represented by a repeating or _____ decimal.
- 27. Problem Solving** Marcus is $5\frac{7}{24}$ feet tall. Ben is $5\frac{5}{16}$ feet tall. Which of the two boys is taller? Justify your answer.
- _____
- 28. Represent Real-World Problems** If one store is selling $\frac{3}{4}$ of a bushel of apples for \$9, and another store is selling $\frac{2}{3}$ of a bushel of apples for \$9, which store has the better deal? Explain your answer.

H.O.T. FOCUS ON HIGHER ORDER THINKING

- 29. Analyze Relationships** You are given a fraction in simplest form. The numerator is not zero. When you write the fraction as a decimal, it is a repeating decimal. Which numbers from 1 to 10 could be the denominator?
- _____
- _____
- 30. Communicate Mathematical Ideas** Julie got 21 of the 23 questions on her math test correct. She got 29 of the 32 questions on her science test correct. On which test did she get a higher score? Can you compare the fractions $\frac{21}{23}$ and $\frac{29}{32}$ by comparing 29 and 21? Explain. How can Julie compare her scores?
- _____
- _____
- _____
- _____
- 31. Look for a Pattern** Look at the decimal 0.121122111222.... If the pattern continues, is this a repeating decimal? Explain.

LESSON 3.2 Adding Rational Numbers

 **FL** 7.NS.1.1d

Apply properties of operations as strategies to add and subtract rational numbers.
Also 7.NS.1.1a, 7.NS.1.1b, 7.NS.1.3



ESSENTIAL QUESTION

How can you add rational numbers?

Adding Rational Numbers with the Same Sign

To add rational numbers with the same sign, apply the rules for adding integers. The sum has the same sign as the sign of the rational numbers.



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EXAMPLE 1

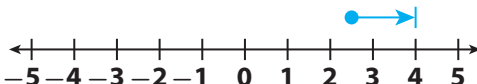


 **FL** 7.NS.1.1b

- A** Malachi hikes for 2.5 miles and stops for lunch. Then he hikes for 1.5 more miles. How many miles did he hike altogether?

STEP 1 Use positive numbers to represent the distance Malachi hiked.

STEP 2 Find $2.5 + 1.5$.

STEP 3 Start at 2.5. 

STEP 4 Move 1.5 units to the *right* because the second addend is *positive*.

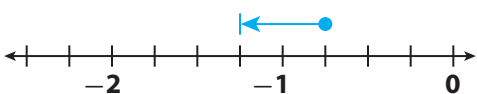
The result is 4.

- Malachi hiked 4 miles.

- B** Kyle pours out $\frac{3}{4}$ liter of liquid from a beaker. Then he pours out another $\frac{1}{2}$ liter of liquid. What is the overall change in the amount of liquid in the beaker?

STEP 1 Use negative numbers to represent amounts the change each time Kyle pours liquid from the beaker.

STEP 2 Find $-\frac{3}{4} + (-\frac{1}{2})$.

STEP 3 Start at $-\frac{3}{4}$. 

STEP 4 Move $|\frac{-1}{2}| = \frac{1}{2}$ unit to the *left* because the second addend is *negative*.

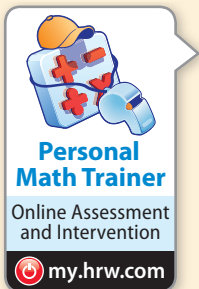
The result is $-1\frac{1}{4}$.

- The amount of liquid in the beaker has decreased by $1\frac{1}{4}$ liters.



Reflect

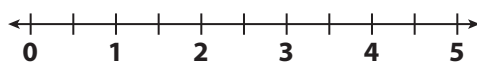
1. Explain how to determine whether to move right or left on the number line when adding rational numbers.



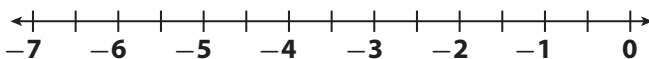
YOUR TURN

Use a number line to find each sum.

2. $3 + 1\frac{1}{2} =$ _____



3. $-2.5 + (-4.5) =$ _____



Adding Rational Numbers with Different Signs

To add rational numbers with different signs, find the difference of their absolute values. Then use the sign of the rational number with the greater absolute value.



EXAMPLE 2



FL 7.NS.1.1b

- A** During the day, the temperature increases by 4.5 degrees. At night, the temperature decreases by 7.5 degrees. What is the overall change in temperature?

STEP 1 Use a positive number to represent the increase in temperature and a negative number to represent a decrease in temperature.

STEP 2 Find $4.5 + (-7.5)$.

STEP 3 Start at 4.5.



STEP 4 Move $|-7.5| = 7.5$ units to the *left* because the second addend is *negative*.

The result is -3 .

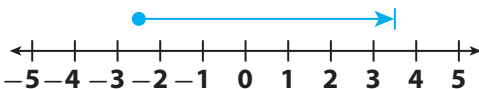
- The temperature decreased by 3 degrees overall.

- B** Ernesto writes a check for \$2.50. Then he deposits \$6 in his checking account. What is the overall increase or decrease in the account balance?

STEP 1 Use a positive number to represent a deposit and a negative number to represent a withdrawal or a check.

STEP 2 Find $-2.5 + 6$.

STEP 3 Start at -2.5 .



STEP 4 Move $|6| = 6$ units to the *right* because the second addend is *positive*.

The result is 3.5.

- The account balance will increase by \$3.50.

Reflect

4. Do $-3 + 2$ and $2 + (-3)$ have the same sum? Does it matter if the negative number is the first addend or the second addend?

5. **Make a Conjecture** Do you think the sum of a negative number and a positive number will always be negative? Explain your reasoning.

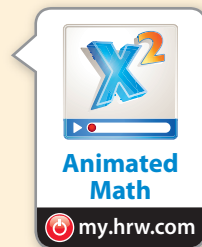
YOUR TURN

Use a number line to find each sum.

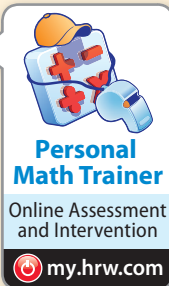
6. $-8 + 5 =$ _____

7. $\frac{1}{2} + \left(-\frac{3}{4}\right) =$ _____

8. $-1 + 7 =$ _____



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Finding the Additive Inverse

The **opposite**, or **additive inverse**, of a number is the same distance from 0 on a number line as the original number, but on the other side of 0. The sum of a number and its additive inverse is 0. Zero is its own additive inverse.

EXAMPLE 3



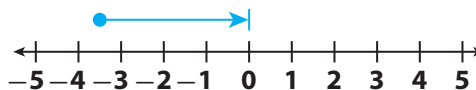
FL 7.NS.1.1a, 7.NS.1.1b, 7.NS.1.1d

A A football team loses 3.5 yards on their first play. On the next play, they gain 3.5 yards. What is the overall increase or decrease in yards?

STEP 1 Use a positive number to represent the gain in yards and a negative number to represent the loss in yards.

STEP 2 Find $-3.5 + 3.5$.

STEP 3 Start at -3.5 .



STEP 4 Move $|3.5| = 3.5$ units to the *right*, because the second addend is *positive*.

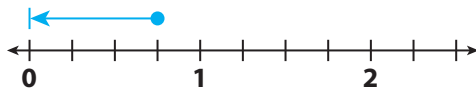
The result is 0. This means the overall change is 0 yards.

B Kendrick adds $\frac{3}{4}$ cup of chicken stock to a pot. Then he takes $\frac{3}{4}$ cup of stock out of the pot. What is the overall increase or decrease in the amount of chicken stock in the pot?

STEP 1 Use a positive number to represent chicken stock added to the pot and a negative number to represent chicken stock taken out of the pot.

STEP 2 Find $\frac{3}{4} + \left(-\frac{3}{4}\right)$.

STEP 3 Start at $\frac{3}{4}$.



STEP 4 Move $\left|-\frac{3}{4}\right| = \frac{3}{4}$ units to the *left* because the second addend is *negative*.

The result is 0. This means the overall change is 0 cups.

Math Talk

Mathematical Practices

Explain how to use a number line to find the additive inverse, or opposite, of -3.5 .

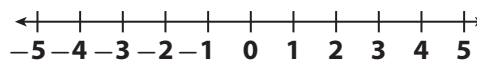
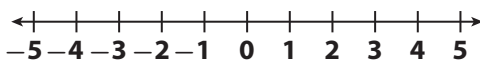
My Notes

YOUR TURN

Use a number line to find each sum.

9. $2\frac{1}{2} + \left(-2\frac{1}{2}\right) =$ _____

10. $-4.5 + 4.5 =$ _____



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Adding Three or More Rational Numbers

Recall that the Associative Property of Addition states that you are adding more than two numbers, you can group any of the numbers together. This property can help you add numbers with different signs.



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EXAMPLE 4



FL 7.NS.1.1d, 7.NS.1.3

Tina spent \$5.25 on craft supplies to make friendship bracelets. She made \$3.75 on Monday. On Tuesday, she sold an additional \$4.50 worth of bracelets. What was Tina's overall profit or loss?

STEP 1 Use *negative* numbers to represent the amount Tina *spent* and *positive* numbers to represent the money Tina *earned*.

STEP 2 Find $-5.25 + 3.75 + 4.50$.

STEP 3 Group numbers with the same sign.

$$-5.25 + (3.75 + 4.50) \quad \text{Associative Property}$$

STEP 4 $-5.25 + 8.25$

Add the numbers inside the parentheses.

Find the difference of the absolute values: $8.25 - 5.25$.

3

Use the sign of the number with the greater absolute value. The sum is positive.

Tina earned a profit of \$3.00.

Profit means the difference between income and costs is positive.



YOUR TURN

Find each sum.

11. $-1.5 + 3.5 + 2 =$ _____

12. $3\frac{1}{4} + (-2) + (-2\frac{1}{4}) =$ _____

13. $-2.75 + (-3.25) + 5 =$ _____

14. $15 + 8 + (-3) =$ _____



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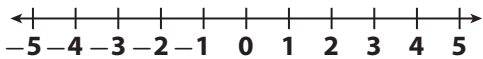
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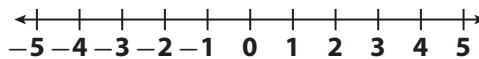
Guided Practice

Use a number line to find each sum. (Example 1 and Example 2)

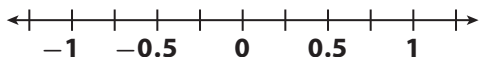
1. $-3 + (-1.5) =$ _____



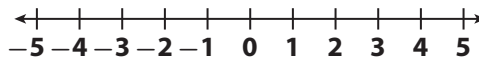
2. $1.5 + 3.5 =$ _____



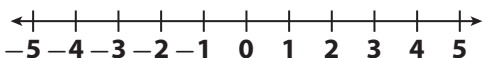
3. $\frac{1}{4} + \frac{1}{2} =$ _____



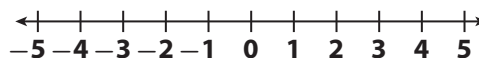
4. $-1\frac{1}{2} + (-1\frac{1}{2}) =$ _____



5. $3 + (-5) =$ _____



6. $-1.5 + 4 =$ _____



7. Victor borrowed \$21.50 from his mother to go to the theater. A week later, he paid her \$21.50 back. How much does he still owe her? (Example 3)

8. Sandra used her debit card to buy lunch for \$8.74 on Monday. On Tuesday, she deposited \$8.74 back into her account. What is the overall increase or decrease in her bank account? (Example 3)

Find each sum without using a number line. (Example 4)

9. $2.75 + (-2) + (-5.25) =$ _____

10. $-3 + (1\frac{1}{2}) + (2\frac{1}{2}) =$ _____

11. $-12.4 + 9.2 + 1 =$ _____

12. $-12 + 8 + 13 =$ _____

13. $4.5 + (-12) + (-4.5) =$ _____

14. $\frac{1}{4} + (-\frac{3}{4}) =$ _____

15. $-4\frac{1}{2} + 2 =$ _____

16. $-8 + (-1\frac{1}{8}) =$ _____




ESSENTIAL QUESTION CHECK-IN

17. How can you use a number line to find the sum of -4 and 6 ?

3.2 Independent Practice



FL 7.NS.1.1a, 7.NS.1.1b, 7.NS.1.1d, 7.NS.1.3



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18. Samuel walks forward 19 steps. He represents this movement with a positive 19. How would he represent the opposite of this number? _____
19. Julia spends \$2.25 on gas for her lawn mower. She earns \$15.00 mowing her neighbor's yard. What is Julia's profit? _____
20. A submarine submerged at a depth of -35.25 meters dives an additional 8.5 meters. What is the new depth of the submarine? _____
21. Renee hiked for $4\frac{3}{4}$ miles. After resting, Renee hiked back along the same route for $3\frac{1}{4}$ miles. How many more miles does Renee need to hike to return to the place where she started? _____
22. **Geography** The average elevation of the city of New Orleans, Louisiana, is 0.5 m below sea level. The highest point in Louisiana is Driskill Mountain at about 163.5 m higher than New Orleans. How high is Driskill Mountain? _____
23. **Problem Solving** A contestant on a game show has 30 points. She answers a question correctly to win 15 points. Then she answers a question incorrectly and loses 25 points. What is the contestant's final score?

Financial Literacy Use the table for 24–26. Kameh owns a bakery. He recorded the bakery income and expenses in a table.

Month	Income (\$)	Expenses (\$)
January	1,205	1,290.60
February	1,183	1,345.44
March	1,664	1,664.00
June	2,413	2,106.23
July	2,260	1,958.50
August	2,183	1,845.12

24. In which months were the expenses greater than the income? Name the month and find how much money was lost. _____
25. In which months was the income greater than the expenses? Name the months and find how much money was gained.

26. **Communicate Mathematical Ideas** If the bakery started with an extra \$250 from the profits in December, describe how to use the information in the table to figure out the profit or loss of money at the bakery by the end of August. Then calculate the profit or loss.

27. **Vocabulary** -2 is the _____ of 2.
28. The basketball coach made up a game to play where each player takes 10 shots at the basket. For every basket made, the player gains 10 points. For every basket missed, the player loses 15 points.

a. The player with the highest score sank 7 baskets and missed 3. What was the highest score?

b. The player with the lowest score sank 2 baskets and missed 8. What was the lowest score?

c. Write an expression using addition to find out what the score would be if a player sank 5 baskets and missed 5 baskets.



FOCUS ON HIGHER ORDER THINKING

29. **Communicate Mathematical Ideas** Explain the different ways it is possible to add two rational numbers and get a negative number.

30. **Explain the Error** A student evaluated $-4 + x$ for $x = -9\frac{1}{2}$ and got an answer of $5\frac{1}{2}$. What might the student have done wrong?

31. **Draw Conclusions** Can you find the sum $[5.5 + (-2.3)] + (-5.5 + 2.3)$ without performing any additions?

Work Area

LESSON 3.3 Subtracting Rational Numbers

 **FL** 7.NS.1.1c

Understand subtraction... as adding the additive inverse... Show that the distance between two rational numbers... is the absolute value of their difference.... Also 7.NS.1.1



ESSENTIAL QUESTION

How do you subtract rational numbers?

Subtracting Positive Rational Numbers

To subtract rational numbers, you can apply the same rules you use to subtract integers.

EXAMPLE 1



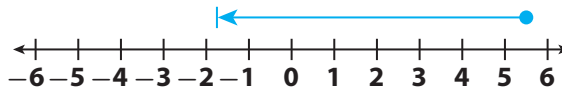
 **FL** 7.NS.1.1

The temperature on an outdoor thermometer on Monday was 5.5°C . The temperature on Thursday was 7.25 degrees less than the temperature on Monday. What was the temperature on Thursday?

Subtract to find the temperature on Thursday.

STEP 1 Find $5.5 - 7.25$.

STEP 2 Start at 5.5 .



STEP 3 Move $|7.25| = 7.25$ units to the *left* because you are subtracting a *positive number*.

The result is -1.75 .

The temperature on Thursday was -1.75°C .

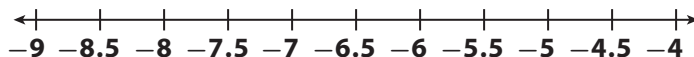


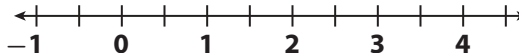
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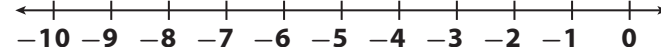
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YOUR TURN

Use a number line to find each difference.

1. $-6.5 - 2 =$ _____ 

2. $1\frac{1}{2} - 2 =$ _____ 

3. $-2.25 - 5.5 =$ _____ 



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Subtracting Negative Rational Numbers

To subtract negative rational numbers, move in the opposite direction on the number line.

EXAMPLE 2



FL 7.NS.1.1

During the hottest week of the summer, the water level of the Muskrat River was $\frac{5}{6}$ foot below normal. The following week, the level was $\frac{1}{3}$ foot below normal. What is the overall change in the water level?

Subtract to find the difference in water levels.

STEP 1 Find $-\frac{1}{3} - (-\frac{5}{6})$.

STEP 2 Start at $-\frac{1}{3}$.



STEP 3 Move $|\frac{5}{6}| = \frac{5}{6}$ to the *right* because you are subtracting a *negative* number.

The result is $\frac{1}{2}$.

So, the water level changed $\frac{1}{2}$ foot.

Reflect

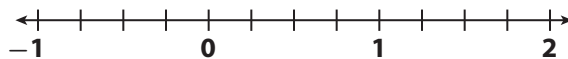
4. Work with other students to compare addition of negative numbers on a number line to subtraction of negative numbers on a number line.

5. Compare the methods used to solve Example 1 and Example 2.

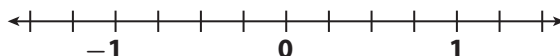
YOUR TURN

Use a number line to find each difference.

6. $0.25 - (-1.50) =$ _____



7. $-\frac{1}{2} - (-\frac{3}{4}) =$ _____



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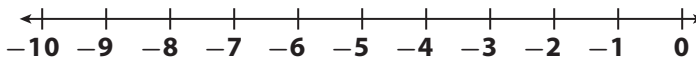
Adding the Opposite

Joe is diving $2\frac{1}{2}$ feet below sea level. He decides to descend $7\frac{1}{2}$ more feet. How many feet below sea level is he?

STEP 1 Use negative numbers to represent the number of feet below sea level.

STEP 2 Find $-2\frac{1}{2} - 7\frac{1}{2}$.

STEP 3 Start at $-2\frac{1}{2}$.



STEP 4 Move $|7\frac{1}{2}| = 7\frac{1}{2}$ units to the _____

because you are subtracting a _____ number.

The result is -10 .

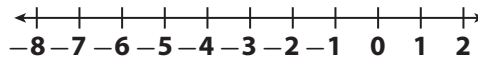
Joe is _____ sea level.

You move left on a number line to add a negative number. You move the same direction to subtract a positive number.

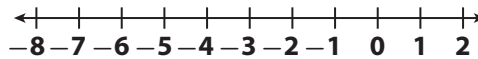
Reflect

8. Use a number line to find each difference or sum.

a. $-3 - 3 =$ _____



b. $-3 + (-3) =$ _____



9. **Make a Conjecture** Work with other students to make a conjecture about how to change a subtraction problem into an addition problem.

Math Talk

Mathematical Practices



Compare the results from 8a and 8b.

Adding the Opposite

To subtract a number, add its opposite. This can also be written as $p - q = p + (-q)$.

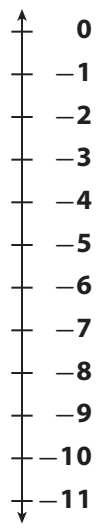
Finding the Distance between Two Numbers

A cave explorer climbed from an elevation of -11 meters to an elevation of -5 meters. What vertical distance did the explorer climb?

There are two ways to find the vertical distance.

- A** Start at _____.
- Count the number of units on the vertical number line up to -5 .
- The explorer climbed _____ meters.
- This means that the vertical distance between -11 meters and -5 meters is _____ meters.

- B** Find the difference between the two elevations and use absolute value to find the distance.
- $-11 - (-5) =$ _____
- Take the absolute value of the difference because distance traveled is always a nonnegative number.
- $|-11 - (-5)| =$ _____
- The vertical distance is _____ meters.



Reflect

- 10.** Does it matter which way you subtract the values when finding distance? Explain.
- _____
- _____
- 11.** Would the same methods work if both the numbers were positive? What if one of the numbers were positive and the other negative?
- _____

Distance Between Two Numbers

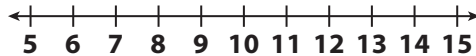
The distance between two values a and b on a number line is represented by the absolute value of the difference of a and b .

Distance between a and $b = |a - b|$ or $|b - a|$.

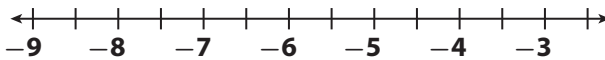
Guided Practice

Use a number line to find each difference. (Example 1, Example 2 and Explore Activity 1)

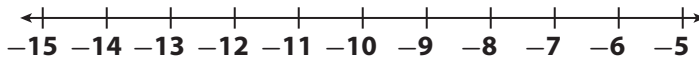
1. $5 - (-8) =$ _____



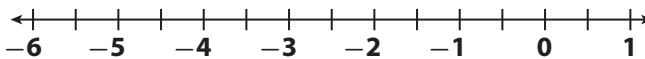
2. $-3\frac{1}{2} - 4\frac{1}{2} =$ _____



3. $-7 - 4 =$ _____



4. $-0.5 - 3.5 =$ _____



Find each difference. (Explore Activity 1)

5. $-14 - 22 =$ _____ 6. $-12.5 - (-4.8) =$ _____ 7. $\frac{1}{3} - (-\frac{2}{3}) =$ _____

8. $65 - (-14) =$ _____ 9. $-\frac{2}{9} - (-3) =$ _____ 10. $24\frac{3}{8} - (-54\frac{1}{8}) =$ _____

11. A girl is snorkeling 1 meter below sea level and then dives down another 0.5 meter. How far below sea level is the girl? (Explore Activity 1) _____

12. The first play of a football game resulted in a loss of $12\frac{1}{2}$ yards. Then a penalty resulted in another loss of 5 yards. What is the total loss or gain? (Explore Activity 1) _____

13. A climber starts descending from 533 feet above sea level and keeps going until she reaches 10 feet below sea level. How many feet did she descend? (Explore Activity 2) _____

14. Eleni withdrew \$45.00 from her savings account. She then used her debit card to buy groceries for \$30.15. What was the total amount Eleni took out of her account? (Explore Activity 1) _____




ESSENTIAL QUESTION CHECK-IN

15. Mandy is trying to subtract $4 - 12$, and she has asked you for help. How would you explain the process of solving the problem to Mandy, using a number line?

3.3 Independent Practice



FL 7.NS.1.1, 7.NS.1.1c



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16. Science At the beginning of a laboratory experiment, the temperature of a substance is -12.6°C . During the experiment, the temperature of the substance decreases 7.5°C . What is the final temperature of the substance?

17. A diver went 25.65 feet below the surface of the ocean, and then 16.5 feet further down, he then rose 12.45 feet. Write and solve an expression to find the diver's new depth.

18. A city known for its temperature extremes started the day at -5 degrees Fahrenheit. The temperature increased by 78 degrees Fahrenheit by midday, and then dropped 32 degrees by nightfall.

a. What expression can you write to find the temperature at nightfall? _____

b. What expression can you write to describe the overall change in temperature? *Hint:* Do not include the temperature at the beginning of the day since you only want to know about how much the temperature changed. _____

c. What is the final temperature at nightfall? What is the overall change in temperature? _____

19. Financial Literacy On Monday, your bank account balance was $-\$12.58$. Because you didn't realize this, you wrote a check for $\$30.72$ for groceries.

a. What is the new balance in your checking account? _____

b. The bank charges a $\$25$ fee for paying a check on a negative balance. What is the balance in your checking account after this fee? _____

c. How much money do you need to deposit to bring your account balance back up to $\$0$ after the fee? _____

Astronomy Use the table for problems 20–21.

20. How much deeper is the deepest canyon on Mars than the deepest canyon on Venus?

Elevations on Planets		
	Lowest (ft)	Highest (ft)
Earth	$-36,198$	29,035
Mars	$-26,000$	70,000
Venus	$-9,500$	35,000

- 21. Persevere in Problem Solving** What is the difference between Earth's highest mountain and its deepest ocean canyon? What is the difference between Mars' highest mountain and its deepest canyon? Which difference is greater? How much greater is it?

- 22.** Pamela wants to make some friendship bracelets for her friends. Each friendship bracelet needs 5.2 inches of string.

- a.** If Pamela has 20 inches of string, does she have enough to make bracelets for 4 of her friends?

- b.** If so, how much string would she have left over? If not, how much more string would she need?

- 23.** Jeremy is practicing some tricks on his skateboard. One trick takes him forward 5 feet, then he flips around and moves backwards 7.2 feet, then he moves forward again for 2.2 feet.

- a.** What expression could be used to find how far Jeremy is from his starting position when he finishes the trick?

- b.** How far from his starting point is he when he finishes the trick? Explain

- 24.** Esteban has \$20 from his allowance. There is a comic book he wishes to buy that costs \$4.25, a cereal bar that costs \$0.89, and a small remote control car that costs \$10.99.

- a.** Does Esteban have enough to buy everything?

- b.** If so, how much will he have left over? If not, how much does he still need?



FOCUS ON HIGHER ORDER THINKING

Work Area

- 25. Look for a Pattern** Show how you could use the Commutative Property to simplify the evaluation of the expression $-\frac{7}{16} - \frac{1}{4} - \frac{5}{16}$.

- 26. Problem Solving** The temperatures for five days in Kaktovik, Alaska, are given below.

-19.6°F , -22.5°F , -20.9°F , -19.5°F , -22.4°F

Temperatures for the following week are expected to be twelve degrees lower every day. What are the highest and lowest temperatures expected for the corresponding 5 days next week?

- 27. Make a Conjecture** Must the difference between two rational numbers be a rational number? Explain.

- 28. Look for a Pattern** Evan said that the difference between two negative numbers must be negative. Was he right? Use examples to illustrate your answer.

LESSON 3.4 Multiplying Rational Numbers

 **FL** 7.NS.1.2

Apply and extend previous understandings of multiplication...and of fractions to multiply...rational numbers. Also 7.NS.1.2a, 7.NS.1.2c



ESSENTIAL QUESTION

How do you multiply rational numbers?

Multiplying Rational Numbers with Different Signs

The rules for the signs of products of rational numbers with different signs are summarized below. Let p and q be rational numbers.

Products of Rational Numbers

Sign of Factor p	Sign of Factor q	Sign of Product pq
+	-	-
-	+	-

You can also use the fact that multiplication is repeated addition.

EXAMPLE 1



 **FL** 7.NS.1.2, 7.NS.1.2a

Gina hiked down a canyon and stopped each time she descended $\frac{1}{2}$ mile to rest. She hiked a total of 4 sections. What is her overall change in elevation?

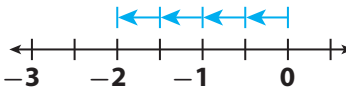
STEP 1 Use a negative number to represent the change in elevation.

STEP 2 Find $4\left(-\frac{1}{2}\right)$.

STEP 3 Start at 0. Move $\frac{1}{2}$ unit to the left 4 times.

The result is -2 .

The overall change is -2 miles.



Check: Use the rules for multiplying rational numbers.

$$4\left(-\frac{1}{2}\right) = \left(-\frac{4}{2}\right) \\ = -2 \checkmark$$

A negative times a positive equals a negative.

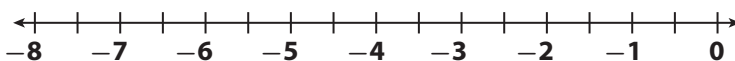
Simplify.



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YOUR TURN

1. Use a number line to find $2(-3.5)$. _____



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Multiplying Rational Numbers with the Same Sign

The rules for the signs of products with the same signs are summarized below.

Products of Rational Numbers		
Sign of Factor p	Sign of Factor q	Sign of Product pq
+	+	+
-	-	+

You can also use a number line to find the product of rational numbers with the same signs.

My Notes

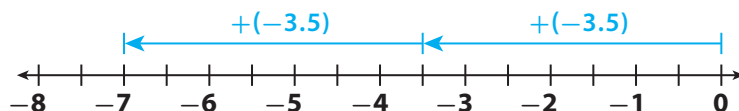
EXAMPLE 2



FL 7.NS.1.2, 7.NS.1.2a

Multiply $-2(-3.5)$.

STEP 1 First, find the product $2(-3.5)$.

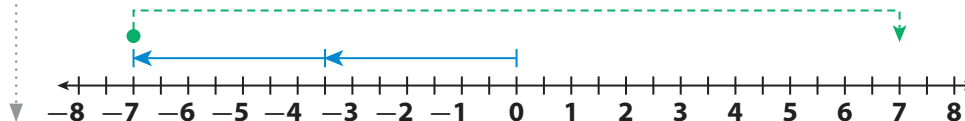


STEP 2 Start at 0. Move 3.5 units to the left two times.

STEP 3 The result is -7 .

STEP 4 This shows that 2 groups of -3.5 equals -7 .

So, -2 groups of -3.5 must equal the *opposite* of -7 .



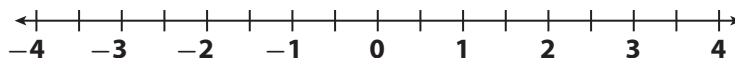
STEP 5 $-2(-3.5) = 7$

Check: Use the rules for multiplying rational numbers.

$-2(-3.5) = 7$ A negative times a negative equals a positive.

YOUR TURN

2. Find $-3(-1.25)$. _____



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Multiplying More Than Two Rational Numbers

If you multiply three or more rational numbers, you can use a pattern to find the sign of the product.



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EXAMPLE 3



FL 7.NS.1.2, 7.NS.1.2c

Multiply $\left(-\frac{2}{3}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{5}\right)$.

STEP 1 First, find the product of the first two factors. Both factors are negative, so their product will be positive.

STEP 2 $\left(-\frac{2}{3}\right)\left(-\frac{1}{2}\right) = +\left(\frac{2}{3} \cdot \frac{1}{2}\right)$
 $= \frac{1}{3}$

STEP 3 Now, multiply the result, which is positive, by the third factor, which is negative. The product will be negative.

STEP 4 $\frac{1}{3}\left(-\frac{3}{5}\right) = \frac{1}{3}\left(-\frac{3}{5}\right)$

STEP 5 $\left(-\frac{2}{3}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{5}\right) = -\frac{1}{5}$

Reflect

3. **Look for a Pattern** You know that the product of two negative numbers is positive, and the product of three negative numbers is negative. Write a rule for finding the sign of the product of n negative numbers.

Math Talk

Mathematical Practices

Suppose you find the product of several rational numbers, one of which is zero. What can you say about the product?

YOUR TURN

Find each product.

4. $\left(-\frac{3}{4}\right)\left(-\frac{4}{7}\right)\left(-\frac{2}{3}\right)$ _____

5. $\left(-\frac{2}{3}\right)\left(-\frac{3}{4}\right)\left(\frac{4}{5}\right)$ _____

6. $\left(\frac{2}{3}\right)\left(-\frac{9}{10}\right)\left(\frac{5}{6}\right)$ _____



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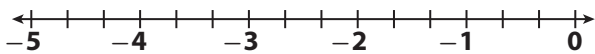
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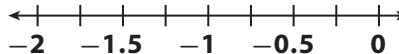
Guided Practice

Use a number line to find each product. (Example 1 and Example 2)

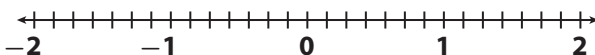
1. $5\left(-\frac{2}{3}\right) =$ _____



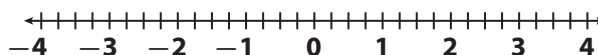
2. $3\left(-\frac{1}{4}\right) =$ _____



3. $-3\left(-\frac{4}{7}\right) =$ _____



4. $-\frac{3}{4}(-4) =$ _____



5. $4(-3) =$ _____

6. $-1.8(5) =$ _____

7. $-2(-3.4) =$ _____

8. $0.54(8) =$ _____

9. $-5(-1.2) =$ _____

10. $-2.4(3) =$ _____

Multiply. (Example 3)

11. $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \square \times \frac{3}{4} =$ _____

12. $-\frac{4}{7}\left(-\frac{3}{5}\right)\left(-\frac{7}{3}\right) = \left(\square\right) \times \left(-\frac{7}{3}\right) =$ _____

13. $-\frac{1}{8} \times 5 \times \frac{2}{3} =$ _____

14. $-\frac{2}{3}\left(\frac{1}{2}\right)\left(-\frac{6}{7}\right) =$ _____

15. The price of one share of Acme Company declined \$3.50 per day for 4 days in a row. What is the overall change in price of one share?

(Example 1)

16. In one day, 18 people each withdrew \$100 from an ATM machine. What is the overall change in the amount of money in the ATM machine?

(Example 1)



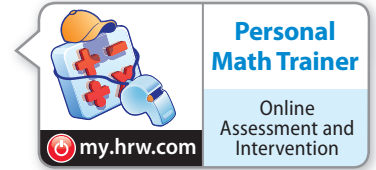
ESSENTIAL QUESTION CHECK-IN

17. Explain how you can find the sign of the product of two or more rational numbers.

3.4 Independent Practice



FL 7.NS.1.2, 7.NS.1.2a, 7.NS.1.2c



- 18. Financial Literacy** Sandy has \$200 in her bank account.
- a. If she writes 6 checks for exactly \$19.98, what expression describes the change in her bank account?
- _____
- b. What is her account balance after the checks are cashed?
- _____
- 19. Communicating Mathematical Ideas** Explain, in words, how to find the product of $-4(-1.5)$ using a number line. Where do you end up?
- _____
- _____
- _____
- _____
- _____
- 20.** Greg sets his watch for the correct time on Wednesday. Exactly one week later, he finds that his watch has lost $3\frac{1}{4}$ minutes. If his watch continues to lose time at the same rate, what will be the overall change in time after 8 weeks?
- _____
- 21.** A submarine dives below the surface, heading downward in three moves. If each move downward was 325 feet, where is the submarine after it is finished diving?
- _____
- _____
- 22. Multistep** For Home Economics class, Sandra has 5 cups of flour. She made 3 batches of cookies that each used 1.5 cups of flour. Write and solve an expression to find the amount of flour Sandra has left after making the 3 batches of cookies.
- _____
- _____
- 23. Critique Reasoning** In class, Matthew stated, "I think that a negative is like an opposite. That is why multiplying a negative times a negative equals a positive. The opposite of negative is positive, so it is just like multiplying the opposite of a negative twice, which is two positives." Do you agree or disagree with his reasoning? What would you say in response to him?
- _____
- _____
- _____
- _____
- _____
- _____
- _____
- _____
- 24.** Kaitlin is on a long car trip. Every time she stops to buy gas, she loses 15 minutes of travel time. If she has to stop 5 times, how late will she be getting to her destination?
- _____
- _____

25. The table shows the scoring system for quarterbacks in Jeremy's fantasy football league. In one game, Jeremy's quarterback had 2 touchdown passes, 16 complete passes, 7 incomplete passes, and 2 interceptions. How many total points did Jeremy's quarterback score?

Quarterback Scoring	
Action	Points
Touchdown pass	6
Complete pass	0.5
Incomplete pass	-0.5
Interception	-1.5



FOCUS ON HIGHER ORDER THINKING

26. **Represent Real-World Problems** The ground temperature at Brigham Airport is 12°C . The temperature decreases by 6.8°C for every increase of 1 kilometer above the ground. What is the overall change in temperature outside a plane flying at an altitude of 5 kilometers above Brigham Airport?

27. **Identify Patterns** The product of four numbers, a , b , c , and d , is a negative number. The table shows one combination of positive and negative signs of the four numbers that could produce a negative product. Complete the table to show the seven other possible combinations.

a	b	c	d
+	+	+	-

28. **Reason Abstractly** Find two integers whose sum is -7 and whose product is 12. Explain how you found the numbers.

Work Area

LESSON 3.5 Dividing Rational Numbers

 **FL** 7.NS.1.2

Apply and extend previous understandings of multiplication and division and of fractions to... divide rational numbers. *Also* 7.NS.1.2b, 7.NS.1.2c



ESSENTIAL QUESTION

How do you divide rational numbers?

EXPLORE ACTIVITY 1



FL 7.NS.1.2, 7.NS.1.2b

Dividing Rational Numbers

A diver needs to descend to a depth of 100 feet below sea level. She wants to do it in 5 equal descents. How far should she travel in each descent?



A To solve this problem, you can set up a division problem: $\frac{-100}{\square} = ?$

B Rewrite the division problem as a multiplication problem. Think: Some number multiplied by 5 equals -100 .

$$\underline{\hspace{2cm}} \times ? = -100$$

C Remember the rules for integer multiplication. If the product is negative, one of the factors must be negative. Since $\underline{\hspace{2cm}}$ is positive, the unknown factor must be **positive / negative.**

D You know that $5 \times \underline{\hspace{2cm}} = 100$. So, using the rules for integer multiplication you can say that $5 \times \underline{\hspace{2cm}} = -100$.

The diver should descend $\underline{\hspace{2cm}}$ feet in each descent.

Reflect

1. What do you notice about the quotient of two rational numbers with different signs?

2. What do you notice about the quotient of two rational numbers with the same sign? Does it matter if both signs are positive or both are negative?

EXPLORE ACTIVITY 1 (cont'd)

Let p and q be rational numbers.

Quotients of Rational Numbers		
Sign of Dividend p	Sign of Divisor q	Sign of Quotient $\frac{p}{q}$
+	-	-
-	+	-
+	+	+
-	-	+

Also, $-\left(\frac{p}{q}\right) = \frac{-p}{q} = \frac{p}{-q}$, for q not zero.

EXPLORE ACTIVITY 2

FL 7.NS.1.2b

Placement of Negative Signs in Quotients

Quotients can have negative signs in different places.

Are the rational numbers $\frac{12}{-4}$, $\frac{-12}{4}$, and $-\left(\frac{12}{4}\right)$ equivalent?

- A** Find each quotient. Then use the rules you found in Explore Activity 1 to make sure the sign of the quotient is correct.

$$\frac{12}{-4} = \underline{\hspace{2cm}} \quad \frac{-12}{4} = \underline{\hspace{2cm}} \quad -\left(\frac{12}{4}\right) = \underline{\hspace{2cm}}$$

- B** What do you notice about each quotient?

- C** The rational numbers **are / are not** equivalent.

- D Conjecture** Explain how the placement of the negative sign in the rational number affects the sign of the quotients.

Reflect

Write two equivalent expressions for each quotient.

3. $\frac{14}{-7}$ _____, _____ 4. $\frac{-32}{-8}$ _____, _____

Quotients of Rational Numbers

The rules for dividing rational numbers are the same as dividing integers.



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EXAMPLE 1



FL 7.NS.1.2c

- A** Over 5 months, Carlos wrote 5 checks for a total of \$323.75 to pay for his cable TV service. His cable bill is the same amount each month. What was the change in Carlos' bank account each month to pay for cable?

Find the quotient: $\frac{-323.75}{5}$

STEP 1 Use a negative number to represent the withdrawal from his account each month.

STEP 2 Find $\frac{-323.75}{5}$.

STEP 3 Determine the sign of the quotient.

The quotient will be negative because the signs are different.

STEP 4 Divide.

$$\frac{-323.75}{5} = -64.75$$

○ Carlos withdrew \$64.75 each month to pay for cable TV.

- B** Find $\frac{\frac{7}{10}}{-\frac{1}{5}}$.

STEP 1 Determine the sign of the quotient.
The quotient will be negative because the signs are different.

STEP 2 Write the complex fraction as division: $\frac{\frac{7}{10}}{-\frac{1}{5}} = \frac{7}{10} \div -\frac{1}{5}$

STEP 3 Rewrite using multiplication: $\frac{7}{10} \times \left(-\frac{5}{1}\right)$ *Multiply by the reciprocal.*

STEP 4 $\frac{7}{10} \times \left(-\frac{5}{1}\right) = -\frac{35}{10}$ *Multiply.*

$= -\frac{7}{2}$ *Simplify.*

$$\frac{\frac{7}{10}}{-\frac{1}{5}} = -\frac{7}{2}$$

YOUR TURN

Find each quotient.

5. $\frac{2.8}{-4} =$ _____

6. $\frac{-\frac{5}{8}}{\frac{6}{7}} =$ _____

7. $-\frac{5.5}{0.5} =$ _____



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Guided Practice

Find each quotient. (Explore Activities 1 and 2, Example 1)

1. $\frac{0.72}{-0.9} =$ _____

2. $\left(-\frac{1}{5}\right) \div \left(-\frac{7}{5}\right) =$ _____

3. $\frac{56}{-7} =$ _____

4. $\frac{251}{4} \div \left(-\frac{3}{8}\right) =$ _____

5. $\frac{75}{-\frac{1}{5}} =$ _____

6. $\frac{-91}{-13} =$ _____

7. $\frac{-\frac{3}{7}}{\frac{9}{4}} =$ _____

8. $-\frac{12}{0.03} =$ _____

9. A water pail in your backyard has a small hole in it. You notice that it has drained a total of 3.5 liters in 4 days. What is the average change in water volume each day? (Example 1)

10. The price of one share of ABC Company declined a total of \$45.75 in 5 days. What was the average change of the price of one share per day? (Example 1)

11. To avoid a storm, a passenger-jet pilot descended 0.44 mile in 0.8 minute. What was the plane's average change of altitude per minute? (Example 1)



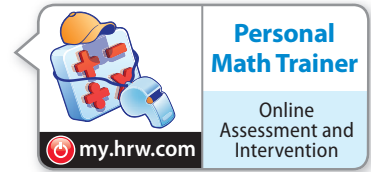
ESSENTIAL QUESTION CHECK-IN

12. Explain how you would find the sign of the quotient $\frac{32 \div (-2)}{-16 \div 4}$.

3.5 Independent Practice



FL 7.NS.1.2, 7.NS.1.2b, 7.NS.1.2c



13. $\frac{5}{-2} \div \frac{-8}{1} =$ _____

14. $5\frac{1}{3} \div (-1\frac{1}{2}) =$ _____

15. $\frac{-120}{-6} =$ _____

16. $\frac{-\frac{4}{5}}{-\frac{2}{3}} =$ _____

17. $1.03 \div (-10.3) =$ _____

18. $\frac{-0.4}{80} =$ _____

19. $1 \div \frac{9}{5} =$ _____

20. $\frac{-\frac{1}{4}}{\frac{23}{24}} =$ _____

21. $\frac{-10.35}{-2.3} =$ _____

22. Alex usually runs for 21 hours a week, training for a marathon. If he is unable to run for 3 days, describe how to find out how many hours of training time he loses, and write the appropriate integer to describe how it affects his time.

23. The running back for the Bulldogs football team carried the ball 9 times for a total loss of $15\frac{3}{4}$ yards. Find the average change in field position on each run.

24. The 6:00 a.m. temperatures for four consecutive days in the town of Lincoln were -12.1°C , -7.8°C , -14.3°C , and -7.2°C . What was the average 6:00 a.m. temperature for the four days?

25. **Multistep** A seafood restaurant claims an increase of \$1,750.00 over its average profit during a week where it introduced a special of baked clams.

- a. If this is true, how much extra profit did it receive per day?

- b. If it had, instead, lost \$150 per day, how much money would it have lost for the week?

- c. If its total loss was \$490 for the week, what was its average daily change?

26. A hot air balloon descended 99.6 meters in 12 seconds. What was the balloon's average rate of descent in meters per second?



27. Sanderson is having trouble with his assignment. His shown work is as follows:

$$\frac{-\frac{3}{4}}{\frac{4}{3}} = -\frac{3}{4} \times \frac{4}{3} = -\frac{12}{12} = -1$$

However, his answer does not match the answer that his teacher gives him. What is Sanderson's mistake? Find the correct answer.

28. **Science** Beginning in 1996, a glacier lost an average of 3.7 meters of thickness each year. Find the total change in its thickness by the end of 2012.



FOCUS ON HIGHER ORDER THINKING

29. **Represent Real-World Problems** Describe a real-world situation that can be represented by the quotient $-85 \div 15$. Then find the quotient and explain what the quotient means in terms of the real-world situation.

30. **Construct an Argument** Divide 5 by 4. Is your answer a rational number? Explain.

31. **Critical Thinking** Should the quotient of an integer divided by a nonzero integer always be a rational number? Why or why not?

Work Area

LESSON 3.6 Applying Rational Number Operations

 **FL** 7.EE.2.3

Solve ... problems ... with positive and negative rational numbers in any form ... using tools strategically. Also 7.NS.1.3



ESSENTIAL QUESTION

How do you use different forms of rational numbers and strategically choose tools to solve problems?

Assessing Reasonableness of Answers

Even when you understand how to solve a problem, you might make a careless solving error. You should always check your answer to make sure that it is reasonable.



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EXAMPLE 1



FL 7.EE.2.3, 7.NS.1.3

Jon is hanging a picture. He wants to center it horizontally on the wall. The picture is $32\frac{1}{2}$ inches long, and the wall is $120\frac{3}{4}$ inches long. How far from each edge of the wall should he place the picture?

STEP 1 Find the total length of the wall not covered by the picture.

$$120\frac{3}{4} - 32\frac{1}{2} = 88\frac{1}{4} \text{ in.}$$

Subtract the whole number parts and then the fractional parts.

STEP 2 Find the length of the wall on each side of the picture.

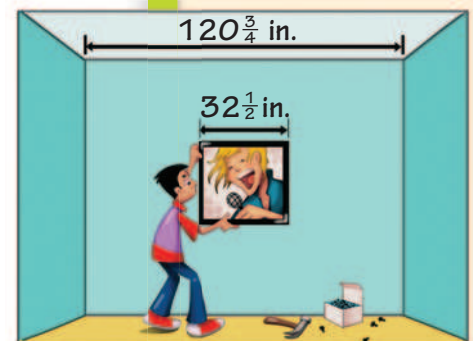
$$\frac{1}{2} \left(88\frac{1}{4} \right) = 44\frac{1}{8} \text{ in.}$$

Jon should place the picture $44\frac{1}{8}$ inches from each edge of the wall.

STEP 3 Check the answer for reasonableness.

The wall is about 120 inches long. The picture is about 30 inches long. The length of wall space left for *both* sides of the picture is about $120 - 30 = 90$ inches. The length left for *each* side is about $\frac{1}{2}(90) = 45$ inches.

The answer is reasonable because it is close to the estimate.



YOUR TURN

1. A 30-minute TV program consists of three commercials, each $2\frac{1}{2}$ minutes long, and four equal-length entertainment segments. How long is each entertainment segment? _____



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Using Rational Numbers in Any Form

You have solved problems using integers, positive and negative fractions, and positive and negative decimals. A single problem may involve rational numbers in two or more of those forms.

EXAMPLE 2

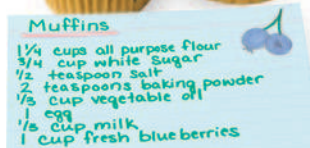
Problem Solving



FL 7.EE.2.3, 7.NS.1.3

My Notes

Alana uses $1\frac{1}{4}$ cups of flour for each batch of blueberry muffins she makes. She has a 5-pound bag of flour that cost \$4.49 and contains seventy-six $\frac{1}{4}$ -cup servings. How many batches can Alana make if she uses all the flour? How much does the flour for one batch cost?



Analyze Information

Identify the important information.

- Each batch uses $1\frac{1}{4}$ cups of flour.
- Seventy-six $\frac{1}{4}$ -cup servings of flour cost \$4.49.

Formulate a Plan

Use logical reasoning to solve the problem. Find the number of cups of flour that Alana has. Use that information to find the number of batches she can make. Use that information to find the cost of flour for each batch.

Solve

Number of cups of flour in bag:

$$76 \text{ cups} \times \frac{1}{4} \text{ cup per serving} = 19 \text{ cups}$$

Number of batches Alana can make:

$$\begin{aligned} \text{total cups of flour} \div \frac{\text{cups of flour}}{\text{batch}} &= 19 \text{ cups} \div \frac{1.25 \text{ cups}}{1 \text{ batch}} \\ &= 19 \div 1.25 \\ &= 15.2 \end{aligned}$$

Write $1\frac{1}{4}$ as a decimal.

Alana cannot make 0.2 batch. The recipe calls for one egg, and she cannot divide one egg into tenths. So, she can make 15 batches.

Cost of flour for each batch: $\$4.49 \div 15 = \0.299 , or about \$0.30.

Justify and Evaluate

A bag contains about 80 quarter cups, or about 20 cups. Each batch uses about 1 cup of flour, so there is enough flour for about 20 batches. A bag costs about \$5.00, so the flour for each batch costs about $\$5.00 \div 20 = \0.25 . The answers are close to the estimates, so the answers are reasonable.

YOUR TURN

2. A 4-pound bag of sugar contains 454 one-teaspoon servings and costs \$3.49. A batch of muffins uses $\frac{3}{4}$ cup of sugar. How many batches can you make if you use all the sugar? What is the cost of sugar for each batch? (1 cup = 48 teaspoons) _____



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Using Tools Strategically

A wide variety of tools are available to help you solve problems. Rulers, models, calculators, protractors, and software are some of the tools you can use in addition to paper and pencil. Choosing tools wisely can help you solve problems and increase your understanding of mathematical concepts.



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EXAMPLE 3



FL 7.EE.2.3, 7.NS.1.3

The depth of Golden Trout Lake has been decreasing in recent years. Two years ago, the depth of the lake was 186.73 meters. Since then the depth has been changing at an average rate of $-1\frac{3}{4}\%$ per year. What is the depth of the lake today?

STEP 1 Convert the percent to a decimal.

$$-1\frac{3}{4}\% = -1.75\% \quad \text{Write the fraction as a decimal.}$$

$$= -0.0175 \quad \text{Move the decimal point two places left.}$$

STEP 2 Find the depth of the lake after one year. Use a calculator to simplify the computations.

$$186.73 \times (-0.0175) \approx -3.27 \text{ meters} \quad \text{Find the change in depth.}$$

$$186.73 - 3.27 = 183.46 \text{ meters} \quad \text{Find the new depth.}$$

STEP 3 Find the depth of the lake after two years.

$$183.46 \times (-0.0175) \approx -3.21 \text{ meters} \quad \text{Find the change in depth.}$$

$$183.46 - 3.21 = 180.25 \text{ meters} \quad \text{Find the new depth.}$$

STEP 4 Check the answer for reasonableness.

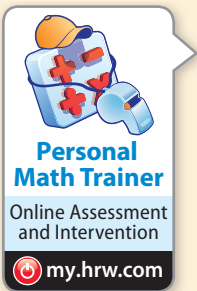
The original depth was about 190 meters. The depth changed by about -2% per year. Because $(-0.02)(190) = -3.8$, the depth changed by about -4 meters per year or about -8 meters over two years. So, the new depth was about 182 meters. The answer is close to the estimate, so it is reasonable.



Math Talk

Mathematical Practices

How could you write a single expression for calculating the depth after 1 year? after 2 years?



YOUR TURN

3. Three years ago, Jolene bought \$750 worth of stock in a software company. Since then the value of her purchase has been increasing at an average rate of $12\frac{3}{5}\%$ per year. How much is the stock worth now? _____

Guided Practice

1. Mike hiked to Big Bear Lake in 4.5 hours at an average rate of $3\frac{1}{5}$ miles per hour. Pedro hiked the same distance at a rate of $3\frac{3}{5}$ miles per hour. How long did it take Pedro to reach the lake? (Example 1 and Example 2)

STEP 1 Find the distance Mike hiked.

$$4.5 \text{ h} \times \boxed{} \text{ miles per hour} = \boxed{} \text{ miles}$$

STEP 2 Find Pedro's time to hike the same distance.

$$\boxed{} \text{ miles} \div \boxed{} \text{ miles per hour} = \boxed{} \text{ hours}$$

2. Until this year, Greenville had averaged 25.68 inches of rainfall per year for more than a century. This year's total rainfall showed a change of $-2\frac{3}{8}\%$ with respect to the previous average. How much rain fell this year? (Example 3)

STEP 1 Use a calculator to find this year's decrease to the nearest hundredth.

$$\boxed{} \text{ inches} \times \boxed{} \approx \boxed{} \text{ inches}$$

STEP 2 Find this year's total rainfall.

$$\boxed{} \text{ inches} - \boxed{} \text{ inches} \approx \boxed{} \text{ inches}$$




ESSENTIAL QUESTION CHECK-IN

3. Why is it important to consider using tools when you are solving a problem?

3.6 Independent Practice



FL 7.NS.1.3, 7.EE.2.3



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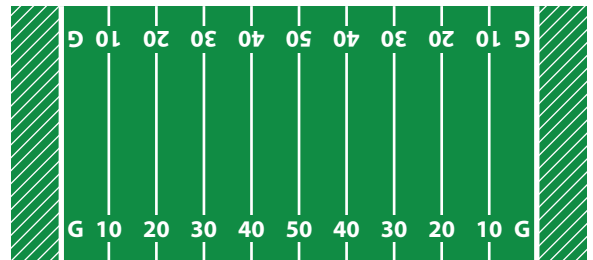
Solve, using appropriate tools.

4. Three rock climbers started a climb with each person carrying 7.8 kilograms of climbing equipment. A fourth climber with no equipment joined the group. The group divided the total weight of climbing equipment equally among the four climbers. How much did each climber carry? _____
5. Foster is centering a photo that is $3\frac{1}{2}$ inches wide on a scrapbook page that is 12 inches wide. How far from each side of the page should he put the picture? _____
6. Diane serves breakfast to two groups of children at a daycare center. One box of Oaties contains 12 cups of cereal. She needs $\frac{1}{3}$ cup for each younger child and $\frac{3}{4}$ cup for each older child. Today's group includes 11 younger children and 10 older children. Is one box of Oaties enough for everyone?



Explain. _____

7. The figure shows how the yard lines on a football field are numbered. The goal lines are labeled G. A referee was standing on a certain yard line as the first quarter ended. He walked $41\frac{3}{4}$ yards to a yard line with the same number as the one he had just left. How far was the referee from the nearest goal line? _____



In 8–10, a teacher gave a test with 50 questions, each worth the same number of points. Donovan got 39 out of 50 questions right. Marci's score was 10 percentage points higher than Donovan's.

8. What was Marci's score? Explain.

9. How many more questions did Marci answer correctly? Explain.

10. Explain how you can check your answers for reasonableness.

For 11–13, use the expression $1.43 \times \left(-\frac{19}{37}\right)$.

11. **Critique Reasoning** Jamie says the value of the expression is close to -0.75 . Does Jamie’s estimate seem reasonable? Explain.

12. Find the product. Explain your method.

13. Does your answer to Exercise 12 justify your answer to Exercise 11?



FOCUS ON HIGHER ORDER THINKING

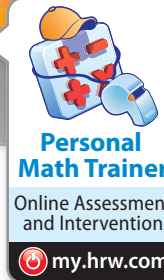
14. **Persevere in Problem Solving** A scuba diver dove from the surface of the ocean to an elevation of $-79\frac{9}{10}$ feet at a rate of -18.8 feet per minute. After spending 12.75 minutes at that elevation, the diver ascended to an elevation of $-28\frac{9}{10}$ feet. The total time for the dive so far was $19\frac{1}{8}$ minutes. What was the rate of change in the diver’s elevation during the ascent? _____

15. **Analyze Relationships** Describe two ways you could evaluate 37% of the sum of $27\frac{3}{5}$ and 15.9. Tell which method you would use and why.

16. **Represent Real-World Problems** Describe a real-world problem you could solve with the help of a yardstick and a calculator.

Work Area

Ready to Go On?



3.1 Rational Numbers and Decimals

Write each mixed number as a decimal.

1. $4\frac{1}{5}$ _____ 2. $12\frac{14}{15}$ _____ 3. $5\frac{5}{32}$ _____

3.2 Adding Rational Numbers

Find each sum.

4. $4.5 + 7.1 =$ _____ 5. $5\frac{1}{6} + (-3\frac{5}{6}) =$ _____

3.3 Subtracting Rational Numbers

Find each difference.

6. $-\frac{1}{8} - (6\frac{7}{8}) =$ _____ 7. $14.2 - (-4.9) =$ _____

3.4 Multiplying Rational Numbers

Multiply.

8. $-4(\frac{7}{10}) =$ _____ 9. $-3.2(-5.6)(4) =$ _____

3.5 Dividing Rational Numbers

Find each quotient.

10. $-\frac{19}{2} \div \frac{38}{7} =$ _____ 11. $\frac{-32.01}{-3.3} =$ _____

3.6 Applying Rational Number Operations

12. Luis bought stock at \$83.60. The next day, the price increased \$15.35. This new price changed by $-4\frac{3}{4}\%$ the following day. What was the final stock price? Is your answer reasonable? Explain.



ESSENTIAL QUESTION

13. How can you use negative numbers to represent real-world problems?



Selected Response

- What is $-7\frac{5}{12}$ written as a decimal?
 - (A) -7.25
 - (B) $-7.333\dots$
 - (C) $-7.41666\dots$
 - (D) -7.512
- Glenda began the day with a golf score of -6 and ended with a score of -10 . Which statement represents her golf score for that day?
 - (A) $-6 - (-10) = 4$
 - (B) $-10 - (-6) = -4$
 - (C) $-6 + (-10) = -16$
 - (D) $-10 + (-6) = -16$
- A submersible vessel at an elevation of -95 feet descends to 5 times that elevation. What is the vessel's new elevation?
 - (A) -475 ft
 - (B) -19 ft
 - (C) 19 ft
 - (D) 475 ft
- The temperature at 7 P.M. at a weather station in Minnesota was -5°F . The temperature began changing at the rate of -2.5°F per hour. What was the temperature at 10 P.M.?
 - (A) -15°F
 - (B) -12.5°F
 - (C) 2.5°F
 - (D) 5°F
- What is the sum of -2.16 and -1.75 ?
 - (A) 0.41
 - (B) 3.91
 - (C) -0.41
 - (D) -3.91

- On Sunday, the wind chill temperature reached -36°F . On Monday, the wind chill temperature only reached $\frac{1}{4}$ of Sunday's wind chill temperature. What was the lowest wind chill temperature on Monday?
 - (A) -9°F
 - (B) $-36\frac{1}{4}^\circ\text{F}$
 - (C) -40°F
 - (D) -144°F
- The level of a lake was 8 inches below normal. It decreased $1\frac{1}{4}$ inches in June and $2\frac{3}{8}$ inches more in July. What was the new level with respect to the normal level?
 - (A) $-11\frac{5}{8}$ in.
 - (B) $-10\frac{5}{8}$ in.
 - (C) $-9\frac{1}{8}$ in.
 - (D) $-5\frac{3}{8}$ in.

Mini-Task

- The average annual rainfall for a town is 43.2 inches.

a. What is the average *monthly* rainfall?

b. The difference of a given month's rainfall from the average monthly rainfall is called the *deviation*. What is the deviation for each month shown?

Town's Rainfall in Last Three Months			
Month	May	June	July
Rain (in.)	$2\frac{3}{5}$	$\frac{7}{8}$	$4\frac{1}{4}$

c. The average monthly rainfall for the previous 9 months was 4 inches. Did the town exceed its average annual rainfall? If so, by how much?
